Calculus Warm Up #5-1

Test for convergence using a test from the side board.

State the test(s) used and give a complete conclusion.

$$\sum_{n=1}^{\infty} \left| \frac{\cos n}{2^n} \right|$$

HW Questions: p. 604

9) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!}$

$$\lim_{n\to\infty} \frac{2^n}{n!}$$

$$= \lim_{n\to\infty} \left[\frac{2^{n+1}}{(n+1)!} \cdot \frac{n!}{2^n} \right]$$

$$\frac{n!}{(n+1)!} = \frac{n(n-1)(n-2)(n-3)(n-4)\ldots}{(n+1)(n+1-1)(n+1-2)(n+1-3)(n+1-4)}$$

$$= \frac{\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}\cancel{(n-4)}\ldots}{(n+1)\cancel{n}\cancel{(n-1)}\cancel{(n-2)}\cancel{(n-3)}\ldots}$$

$$= \frac{1}{n+1}$$

15) $\sum_{n=0}^{\infty} \frac{3^n}{(n+1)^n}$ $\lim_{n\to\infty}\left[\frac{3^{n+1}}{(n+2)^{n+1}} \cdot \frac{(n+1)^n}{3^n}\right]$

$= \lim_{n\to\infty}\left(\frac{3}{n+2}\right) \cdot \lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^n$

$= 0 \cdot \frac{1}{e}$

$= 0 < 1$ converges

☆ See next slide for how to evaluate $\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^n$

$\lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^n$ let $y = \left(\frac{n+1}{n+2}\right)^n$

$\ln y = n \ln\left(\frac{n+1}{n+2}\right)$

$\ln y = \lim_{n\to\infty}\left[\frac{\ln(n+1) - \ln(n+2)}{\frac{1}{n}}\right] = \frac{0}{0}$

$\ln y = \lim_{n\to\infty} \frac{\left(\frac{1}{n+1} - \frac{1}{n+2}\right)}{-\frac{1}{n^2}}$

$\ln y = \lim_{n\to\infty}\left(\frac{n+2 - (n+1)}{n^2+3n+2} \cdot \frac{-n^2}{1}\right.$

$\ln y = \lim_{n\to\infty} \frac{-n^2}{n^2+3n+2}$

$\ln y = -1$

$y = \frac{1}{e}$ $\longrightarrow$ $\therefore \lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^n = \frac{1}{e}$

17) $\sum_{n=0}^{\infty} \frac{4^n}{3^n+1}$

$$\lim_{n\to\infty}\left[\frac{4^{n+1}}{3^{n+1}+1}\cdot\frac{3^n+1}{4^n}\right]$$

$$=\lim_{n\to\infty}\frac{4(3^n+1)}{3^{n+1}+1}\cdot\frac{\frac{1}{3^n}}{\frac{1}{3^n}}$$

$$\frac{1}{3^n}\cdot\frac{3^{n+1}}{1}$$

$$=\lim_{n\to\infty}\left(\frac{4\left(1+\frac{1}{3^n}\right)}{3+\frac{1}{3^n}}\right)$$

$$=\frac{4}{3}>1 \quad \text{diverges.}$$

19) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}n!}{1\cdot 3\cdot 5\cdots(2n+1)}$

$$\lim_{n\to\infty}\left(\frac{(n+1)!}{\cancel{1\cdot 3\cdot 5\cdots(2n+1)}(2(n+1)+1)}\cdot\frac{\cancel{1\cdot 3\cdot 5\cdots(2n+1)}}{n!}\right)$$

$$\lim_{n\to\infty}\frac{n+1}{2n+3}=\frac{1}{2}<1$$

converges.

Classwork: BC MC - A Practice test

*Things we still need to learn:

#7, 10, 17, 26 (#13: derivatives of parametric equations. Try it!)

check answers:

1. D	6. B	11. B	16. E	21. E	26. C
2. C	7. B	12. D	17. D	22. A	27. E
3. C	8. D	13. B	18. B	23. B	28. A
4. E	9. A	14. C	19. D	24. D	
5. D	10. D	15. E	20. E	25. E	

MC - A

2) $p(0) = 4 - \int_0^{\pi/2} \sin(2t)\,dt$

$= 4 - \int_0^{\pi/2} 2 \sin t \cos t \, dt$ $\quad u = \sin t$, $du = \cos t \, dt$

$= 4 - 2\left[\frac{(\sin t)^2}{2}\right]_0^{\pi/2}$

$= 4 - \left[\left(\sin\frac{\pi}{2}\right)^2 - (\sin 0)^2\right]$

$= 4 - 1$

$= 3$ $\quad \boxed{C}$

4) $\lim_{h\to 0} \dfrac{\tan(0+h)-\tan(0)}{h}$

$= \lim_{h\to 0} \dfrac{\tan h - 0}{h} = \dfrac{0}{0}$ so L'Hôp

$= \lim_{h\to 0} \sec^2 h$

E $\sec^2 x$

14) $\int \dfrac{8}{(x+2)(x-2)}$

$\dfrac{8}{(x+2)(x-2)} = \dfrac{A}{x+2} + \dfrac{B}{x-2}$

$8 = A(x-2) + B(x+2)$

for $x=2$ $\quad 8 = B(4) \to B = 2$

for $x=-2$ $\quad 8 = A(-4) \to A = -2$

$= \int \dfrac{-2}{x+2}\,dx + \int \dfrac{2}{x-2}\,dx$

$= -2\ln|x+2| + 2\ln|x-2| + C$

$= 2\left[\ln|x-2| - \ln|x+2|\right] + C$

$= 2\ln\left|\dfrac{x-2}{x+2}\right| + C$

$\boxed{C}$

21) where $f' = 0$ or undef. and goes from − to +

$$f'(x) = \frac{6}{\sqrt[3]{x}} + 3$$

$f' = 0$ @ $x = -8$

undef @ $x = 0$

$-27 \quad -1 \quad 1$

$+ \quad -8 \quad - \quad 0 \quad +$

$\boxed{E}$

22) $\int x f'(x)\,dx$

let $u = x$, $du = 1\,dx$

$dv = f'(x)\,dx$, $v = f(x)$

$$\int x f'(x)\,dx = x \cdot f(x) - \int f(x)\,dx$$

$$@x=4 \rightarrow \int_0^4 x \cdot f'(x)\,dx = 4 \cdot f(4) - \int_0^4 f(x)\,dx$$

$$= 4(-3) - 8$$

$$= -20 \qquad \boxed{A}$$

21) where $f' = 0$ or undef. and goes from − to +

$$f'(x) = \frac{6}{\sqrt[3]{x}} + 3$$

$f' = 0$ @ $x = -8$

undef @ $x = 0$

$-27 \quad -1 \quad 1$

$+ \quad -8 \quad - \quad 0 \quad +$

$\boxed{E}$

22) $\int x f'(x)\,dx$

let $u = x$, $du = 1\,dx$

$dv = f'(x)\,dx$, $v = f(x)$

$$\int x f'(x)\,dx = x \cdot f(x) - \int f(x)\,dx$$

$$@x=4 \rightarrow \int_0^4 x \cdot f'(x)\,dx = 4 \cdot f(4) - \int_0^4 f(x)\,dx$$

$$= 4(-3) - 8$$

$$= -20 \qquad \boxed{A}$$

27) $f'(x) = \frac{d}{dx}\int_4^{2x} \sqrt{t^2 - t}\, dt$

$= \sqrt{(2x)^2 - (2x)} \cdot \frac{d}{dx}[2x]$

$f'(x) = 2\sqrt{4x^2 - 2x}$

$f'(2) = 2\sqrt{12}$

$\boxed{E}$

Classwork & Homework:

BC Practice MC part B

Still need to learn:

\# 78, 86, 87, 90

MC part B answers:

76. B
77. B
78. D
79. E
80. B

81. A
82. C
83. D
84. C
85. B

86. E
87. B
88. C
89. B
90. D
91. B
92. C